# Optical Design of the Rapid Cycling Medical Synchrotron

Javier Cardona, Steve Peggs, and Jorg Kewisch

*Abstract*—The optical design of the rapid cycling medical synchrotron (RCMS) is based on the strong focusing principle. Several advantages arise from this choice of design, including smaller beam sizes and more optimization possibilities. In this paper, a detailed description of the optical design of all beam lines of the facility is presented with the methods that have been used to optimize such beam lines. In particular, we describe a method to match beam lines optimally in one step using as input parameters the beta functions at the matching point. This method eliminates the need of doing repeated simulations varying several parameters of the matching elements until the match is finally achieved.

Comparisons between old designs and the most recent design are presented making special emphasis on cost savings between them. Comparisons with other accelerators and technologies is also presented.

*Index Terms*—FODO, optics, protons, synchrotron.

## I. INTRODUCTION

THE rapid cycling medical synchrotron (RCMS) is a second generation proton therapy synchrotron offering more flexible performance in a simpler lighter and more robust implementation (see, for example, [1] or [2]).

The kinetic energy of the beam extracted from the synchrotron will be between 60 MeV and 250 MeV. A maximum energy of about 215–220 MeV is expected at the patient, if the beam is double scattered in the nozzle.

An energy of 7 MeV has been chosen for injection. This relatively high energy guarantees negligible space charge effects and a relatively small beam size.

The facility will not be capable of full depth proton radiography (unless the maximum energy is raised to about 270 MeV). An energy of 220 MeV has a depth of penetration in tissue of approximately 30 cm, an adequate depth for the treatment of most tumors.

The beam intensity will be about $3 \times 10^{12}$ protons per minute. Although this is small compared to the intensities achieved by some synchrotrons for high-energy physics, still it is not trivial to achieve in a small synchrotron with a low injection energy. The intensity requirement is "soft" in the sense that the highest

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TABLE I
PRIMARY PARAMETERS OF THE RCMS

| | | |
|---|---|---|
| Maximum Extraction Energy | [MeV] | 250 |
| Minimum Extraction Energy | [MeV] | 60 |
| Injection Kinetic Energy | [MeV] | 7 |
| Repetition Rate $f_{rep}$ | [Hz] | 30 |
| Treatment protons per bunch N, min | | $1.0 \times 10^{7}$ |
| Treatment protons per bunch N, max | | $1.7 \times 10^{9}$ |
| Proton Flux R, max | [1/min] | $3.0 \times 10^{12}$ |
| Circumference C | [m] | 28.6 |
| Normalized RMS emittance $\epsilon$ | [μm] | 0.15 |

intensities are needed only occasionally, mainly to treat large tumors in reasonable times (generally in exposure times of order a minute per visit), and the needs are sometimes inflated by allowing for various inefficiencies, especially the losses which inevitably accompany passive beam-spreading techniques. Nevertheless, the RCMS has been design to achieve a higher intensity, at least $3 \times 10^{12}$ protons per minute. The most important consequence of the high design intensity may be that no intensity dependence of the accelerator performance or beam sizes will be encountered in everyday operation as the intensity is adjusted over a range of values well below the ultimate performance limit. Higher intensities may also prove valuable for shortening treatment times and/or for supplying beam to additional treatment rooms in the future. In Table I, we have summarized the basic parameters of the RCMS, some of them already mentioned in this section.

The RCMS will reduce the typical treatment time and at the same time will reduce the risk of dumping a large amount of radiation into the patient. This is possible thanks to the rapid cycling and fast extraction design choices of the accelerator. In contrast, slow cycling demands that a lot of protons occupy the accelerator simultaneously in order to achieve acceptable treatment times; slow extraction thus carries the threat of suddenly dumping a lot of beam into a localized region of a patient. A fast repetition rate allows the intensity requirements to be met with modest intensity per cycle, thereby eliminating intensity-dependent problems such as coherent instabilities and space-charge effects. It also allows scanning of large tumor volumes by delivering one beam bunch to each volume element or voxel, creating the desired dose distribution throughout the tumor while minimizing the dose delivered to normal tissue.

In general terms, the RCMS is composed of one synchrotron ring, four treatment rooms with gantries, and two additional rooms with fixed beam lines (see Fig. 1). The optical design is based on the strong focusing principle: most of the beam lines are built out of a basic optical configuration called FODO cell.

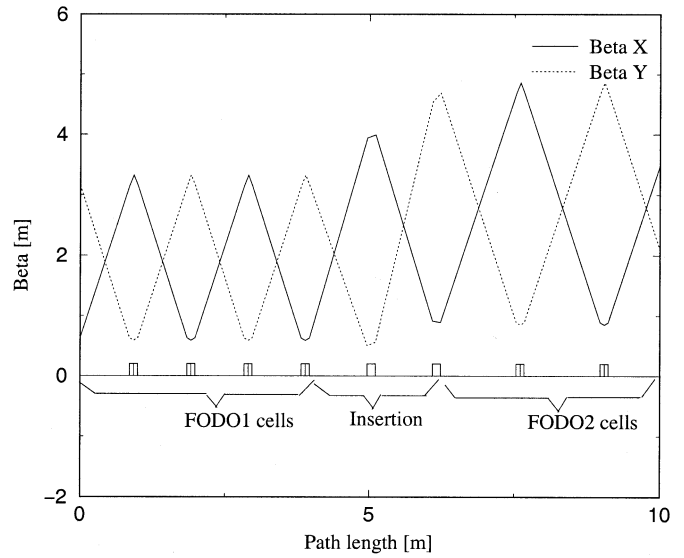Fig. 1.   Physical layout of the RCMS. Only one gantry room is shown.



Fig. 2.   FODO cells structure of different lengths are matched by inserting two quadrupoles between them (the narrow rectangles represent quadrupoles).

The FODO cell is made out of one focusing quadrupole (F), a drift space or dipole bend (O), a defocusing quadrupole (D), and a drift space or dipole bend (O).

The general specifications and design choices of the RCMS has been studied in detail elsewhere [1]–[4]. This article is mainly concerned with the optical design of the facility and the related studies to reduce the cost of the optical lattice.

## II. INCOMING BETA FUNCTION MATCHING TO FODO (IBEFUMFO): THE OPTICS DESIGN STRATEGY

The linear motion of a particle in a beam line is defined by dipole magnets and quadrupole magnets. The motion of any particle is in general an oscillation (a betatron oscillation) around some central trajectory that is defined by the dipole magnets. The betatron oscillations can be mathematically described by

$$z(s) = \sqrt{2J\beta_z(s)} \sin\left(\psi_z(s) - \psi_0\right) \qquad (1)$$

where $z(s)$ can be either of the transverse coordinates $x$ or $y$, $\beta_z(s)$ is the beta function at position $s$ and depends on the particular configuration of quadrupoles in the beam line, $\psi_z(s)$ is the phase advance and $J$, and $\psi_0$ are constants of motion. The beta functions along with the alpha functions defined by

$$\alpha = -\frac{1}{2}\frac{d\beta(s)}{ds} \qquad (2)$$

are called the lattice functions and their calculation is the primary goal in the optical design of a beam line.

A typical problem in optics design is to match beam lines that have different lattice functions at the point where the two beam lines need to be joined. If such a difference is not corrected, the joined beam lines will readjust their lattice functions such that there will be an increase of the maximum beta function.

In some cases, the growth of the beta functions is imperceptible and the overall performance of the whole beam line is not perturbed and hence correction is not needed.

When the growth of the beta functions is not tolerable, correction can be done, for example, by inserting quadrupoles between the two beam lines. The strengths of the quadrupoles and

the distances between them are varied until the lattice functions are matched.

Fig. 2 is a simple example of the application of this method. In this case, the beam lines to be matched are both made out of FODO cells with quadrupoles that have the same focal length but with different spacing between the quadrupoles.

The most common situation found in optical design is to have arbitrary incoming lattice functions that need to be matched to a FODO cell structure. In some cases, the physical parameters (lengths and focusing strengths) of such FODO cells are known beforehand and, hence, the matching can be done by standard algorithms included in simulation software like MAD [5]. In other cases, the physical parameters of the FODO cell are not known before the matching and the only restriction on the final FODO cell is the beam size or the maximum beta functions allowed. The matching in this case can be done in two different ways explained below.

The first possibility is to deduce relations between the physical parameters of the FODO cell and its maximum and minimum beta functions. Simple manipulation of the FODO cell relations found in [6] lead to

$$F = \frac{\sqrt{\beta_F \beta_D}}{2} \qquad (3)$$

and

$$L = \frac{\beta_F - \beta_D}{\beta_F + \beta_D}\sqrt{\beta_F \beta_D}. \qquad (4)$$

Equations (3) and (4) give the focal length $F$ and the length of the FODO cell $L$ as function of the maximum beta function $\beta_F$ and minimum beta function $\beta_D$ of FODO cell as was required initially.

Since all the physical parameters of the FODO cell are known at this point, the problem of doing the matching between a beam line and the FODO cell (see Fig. 2) can now be easily solved with MAD as mentioned before.

The second possibility is to find a relation between the alpha and the beta functions at the matching point of the FODO cell. In this way, once the beta functions are given all the lattice functions will be known at the matching point. The inserted quadrupoles can then be varied to match the lattice functions at the entrance of the FODO cell.

The required relations between the alpha and the beta functions were found to be [3]

$$\alpha_y = \frac{\sqrt{\beta_y}}{\sqrt{\beta_x}}$$
$$\alpha_x = -\frac{\sqrt{\beta_x}}{\sqrt{\beta_y}} \tag{5}$$

where the subindexes $x$ and $y$ stand for the horizontal and the vertical plane, respectively.

Notice than in both cases there are only two inputs parameters which are the beta functions or equivalently the required beam size in the FODO cell. It is also possible to put some other conditions in the design besides the beam size, for example maximizing the length of the FODO cell from (4). In such a case, once the maximum beta function is specified the corresponding minimum beta function will be determined.

This approach in which either (3) and (4) or (5) are used in combination with MAD numerical matching is what we have named Incoming BEta FUnction Matching to FOdo (IBEFUMFO) cells. The IBEFUMFO algorithm were one the most important tools in the optimization of the optical design of the RCMS.

## III. RING OPTICS DESIGN

The synchrotron ring has two arcs and two straight sections. The arcs are built with $D$ and $F$ combined function magnets which are dipole magnets with the poles slightly tilted to add a quadrupole component. The number of dipoles in each arc was determined by the desired beam size and for practical reasons like the symmetry of the lattice.

Dispersion is made equal to zero outside the arcs by adjusting the strength of the focusing quadrupole component of the combined function magnets. However, after the dispersion matching the horizontal and vertical beta functions separated from each other leading to bigger beam sizes. The defocusing strength and the edge angle of the combined function magnets are then used to bring together the horizontal and the vertical beta functions again.

The straight sections should have intervening spaces between the quadrupoles long enough to hold the instrumentation and the utility components. In particular, it is desirable to have a slot for the RF cavities and extraction septum longer than the distance between the centers of two adjacent combined functions magnets in the arcs. This unavoidably leads to an overall change in the beta functions around the ring unless special matching conditions are imposed in the border of the straight sections and the arcs.

The IBEFUMFO algorithm described in Section II can be used in this case to optimally perform the required matching. As stated in Section II, IBEFUMFO uses two quadrupoles to do the matching. The first two quadrupoles of the straight section
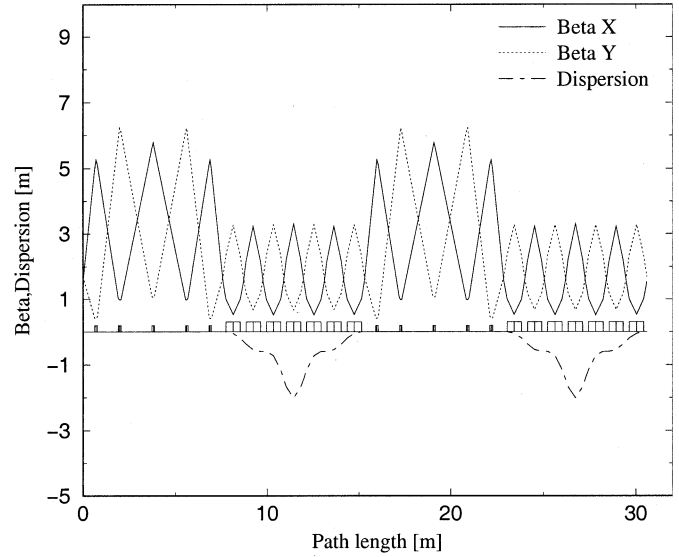


Fig. 3. The optical design of the arcs and the straight sections is done independently. The IBEFUMFO algorithm is then used to match both beam lines. Since arcs are identical and straight sections are also identical the lattice ring can be easily build as shown.

closer to any arc constitute the two required quadrupoles and the central quadrupole of the straight section and the spaces at each side of this quadrupole constitute the final FODO cell structure (see Figs. 2 and 3).

Since arcs and straight sections have mirror symmetry the matching done on one side is identical to the match required on the other side. Also, the straight sections are identical which means that the same power supplies can be used for both straight sections. Only three different quadrupole power supplies are required to do the matching in the whole ring.

Notice that the solution provided by the IBEFUMFO algorithm avoids any increase of the beta functions in the arcs and leads to an acceptable increase of the beta functions in the straight sections (see Fig. 3).

## IV. DELIVERY SYSTEM OPTICS

The delivery system is composed of the extraction beam line, the switch yard beam line, the transport beam line, gantry optical interface, and gantries. The beam is extracted from the synchrotron by a fast kicker followed by a septum magnet. The extraction line comes just after the septum magnet. Since the switch yard is a configuration of FODO cells, IBEFUMFO can be directly applied in this case to make the optical matching between the extraction line and the switch yard beam line. The extraction line is designed such that the output lattice functions comply with (5) and also the dispersion is zero outside the extraction line (see Fig. 4).

Since the gantries all have an identical design it is desirable to have the same lattice functions at the entrance of these beam lines. The switch yard has been designed with this objective in mind as a perfect periodic structure.

The transport lines take the beam from the switch yard to the different rooms of the facility. The research room has two transport lines with bending angles that differ by $30°$. The fixed beam room has one $45°$ transport line that goes to the vertical
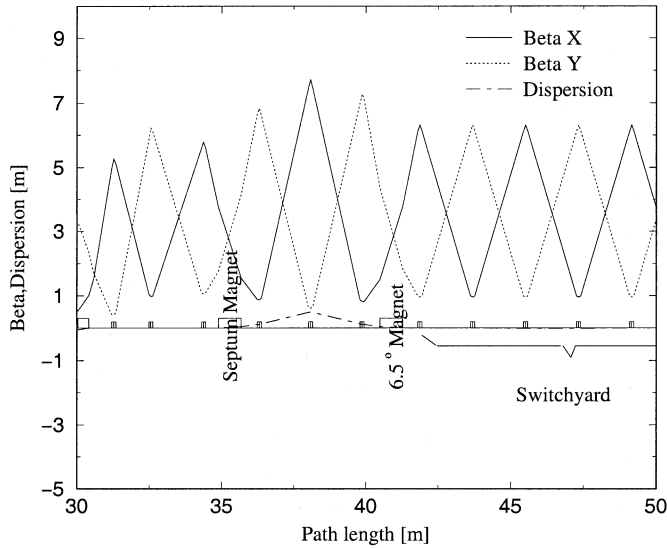
Fig. 4. The interface between the extraction line and the switch yard simultaneously suppresses the dispersion initiated by the septum magnet and smoothly matches the beta functions to the FODO cells structure of the switch yard.
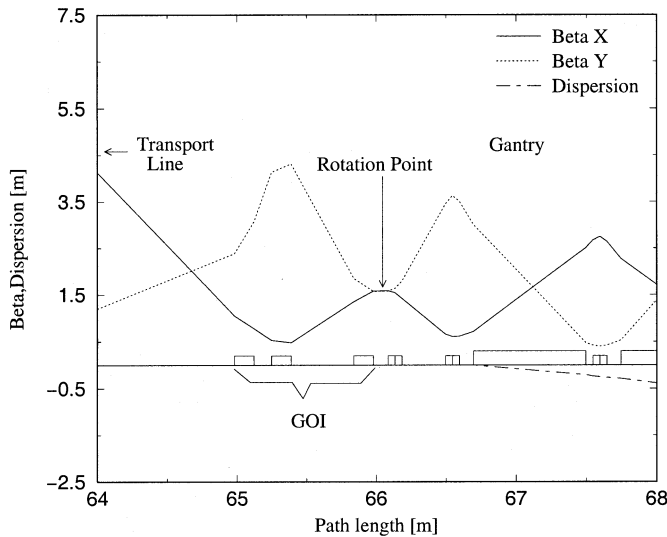


Fig. 5. The distances and strengths of three quadrupoles (Gantry optical interface) before the rotation point are set such that the alpha functions are equal to zero and the beta functions are equal in both planes.

fixed beam line and two additional 90° fixed transport lines. The transport lines that connect the switch yard with the gantry are all identical and the same as the 45° transport lines used in the the fixed beam room.

The gantry requires axially symmetric optics at the point of rotation. A special optical structure that we have named "gantry optical interface" has been designed to achieve this objective by making the horizontal and vertical beta function equal at the point of rotation and the alpha functions equal to zero (see Fig. 5). The matching is done by inserting three quadrupoles between the transport line and the gantry. The distances between the quadrupoles and the strengths of two of them are varied until the matching conditions are satisfied.

For the design of the gantry light weight and compactness are of the highest priority. The gantry dipole deflects the beam by
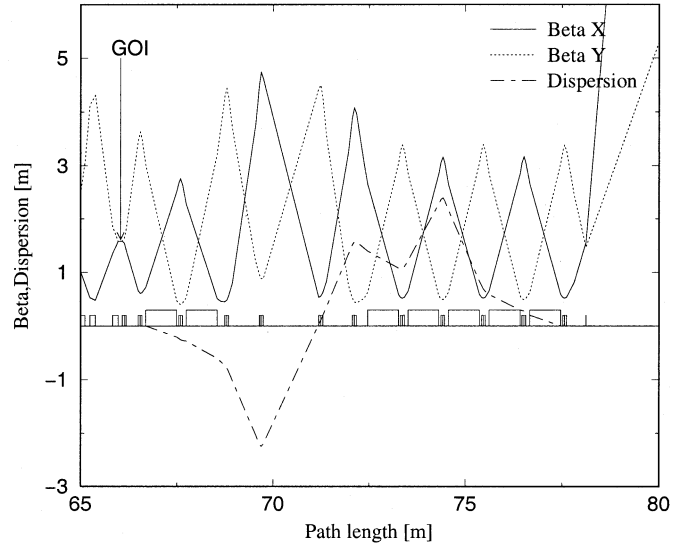


Fig. 6. The achromatic Gantry is built with seven 30° dipole. Dispersion is match such that it is zero at the gantry output.

TABLE II
COMPARISON BETWEEN THE NUMBER OF MAGNETS NEEDED BEFORE AND AFTER DOING THE TRAJECTORY STUDIES. THOSE STUDIES SHOW THAT C-TYPE MAGNETS AND 6.5° MAGNET CAN BE REPLACED WITH THE 22.5° CHEVRON MAGNET

| Magnet Style | No (Before) | No (After) |
|---|---|---|
| 25.71° Chevrons | 0 | 14 |
| 22.5° Chevrons | 16 | 21 |
| H-type (gantry) | 35 | 35 |
| H-type (6.5°) | 1 | 0 |
| C-type | 22 | 0 |
| O-type (Hor) | 4 | 4 |
| O-type (Ver) | 4 | 4 |

30° instead of 22.5°, maximizing the "packing factor" (ratio of integrated dipole length to the total length). Seven 30° dipoles are used to built the gantry and only four quadrupole power supplies. Lattice functions of the gantry are shown in Fig. 6.

## V. MAGNET CONSIDERATIONS

The RCMS originally was designed to have 6 different dipole magnets styles (see Table II): a Chevron magnet used in the synchrotron ring, a C-type magnet used in the switch yard and transport lines, an H-type rectangular magnet used in the gantry, an H-type rectangular magnet used as a 6.5° magnet in the extraction line, and two O-type rectangle magnet for the horizontal and vertical correctors respectively. There are also two different styles of quadrupoles, one for the synchrotron and other for the gantry.

The Chevron magnet is a special kind of dipole "built" out of two rectangular bends. This procedure reduces the maximum excursion of the beam (beam sagitta) within the beam pipe and hence it is possible to use smaller beam pipes [7].

In order to reduce costs, trajectory studies were done to determine the feasibility of reducing the number of dipole styles. In
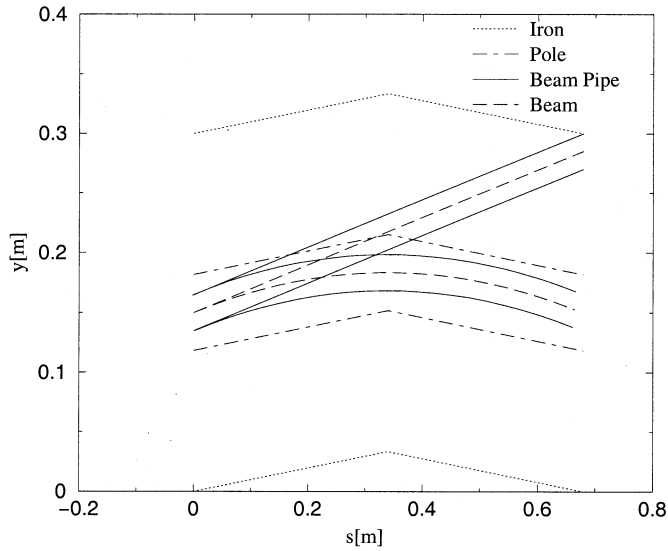
Fig. 7.   The trajectories and beam pipes of the switch yard magnet within the iron of the 22.5° dipole showing that the beam can go through the magnet even when it is off.
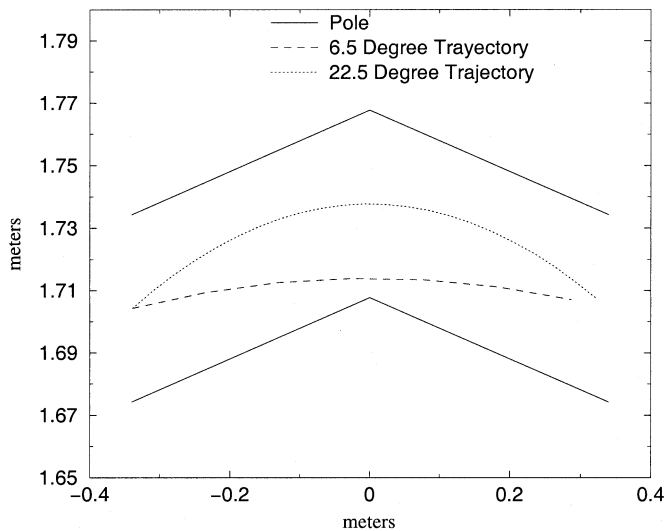


Fig. 8.   The trajectory of the beam when the 22.5° magnet is used as 6.5° magnet. The 6.5° trajectory fits loosely within the pipe of the magnet.

particular, the possibility of using the 22.5° Chevrons as switch yard magnets (C-type) is studied in Fig. 7 which shows the beam pipe and the beam itself when the magnet is off (straight beam pipe) and the corresponding pictures when the magnet is on (curved beam pipe) over the magnet iron. The pipes fit exactly within the iron of the synchrotron dipole allowing this dipole to be used as switch yard dipole also.

Similar studies have shown that it is also possible to use a 22.5° synchrotron dipole as the 6.5° dipole (see Fig. 8) needed in the extraction line.

The new synchrotron dipole is a combined function magnet with a bending angle of 25.71° and the same magnet is also a chevron magnet as in the old version.

In summary, the number of styles of magnets has been reduced from 6 to 5 (see Table II) which has a direct impact on the cost of the facility not only in the design stage but also during commissioning and normal operation.

TABLE  III
COST CHANGE BETWEEN SOME OF THE SYSTEMS OF THE OLD DESIGN OF THE RCMS AND THE CORRESPONDING ONES TO THE MOST RECENT DESIGN

| System | Cost Change |
|---|---|
| Magnets | - 22.6 % |
| Inj. & Extr. | -10 % |
| Power Supplies | -6.5 % |
| Radio Frequency Cavities | -50 % |

## VI.  COMPARISONS WITH PREVIOUS DESIGNS

The previous versions of the RCMS used separated magnets for the dipoles and quadrupoles in the arcs of the ring. The introduction of combined function magnets in the new design has saved 16 quadrupoles.

The switch yard was also redesigned with the aid of the IBEFUMFO algorithm leading to a perfectly periodic structure with a reduced number of quadrupole. The total number of quadrupoles in the delivery system went down from 175 quadrupoles to 139 quadrupoles.

Taking into account the above considerations, the total cost of the design and fabrication of all magnets has dropped by 22.6% as can be seen on Table III.

Since the total number of magnets has been reduced, the requirements on the power supplies are less and, hence, it has been possible to lower their fabrication cost by 6.5% as shown on Table III.

The RF system in the previous design were composed of two one gap cavities powered by solid state amplifiers. Since the straight sections now have a longer space available (1.8 m) it was possible to design a single cavity of two gaps to provide the neccesary energy to the particles each turn. This modification on the RF system then lower the fabrication cost of the cavities by 50% as indicated on Table III.

The injection and extraction system have also benefited from the longer straight sections. Since the electrostatic inflector used at injection and the septum magnet used at extraction are now longer the voltage required for the inflector and the magnetic field required for the septum magnet have each been reduced by about one third. This reduces the cost of these systems in the amount listed in Table III.

## VII.  COMPARISONS WITH OTHER MEDICAL ACCELERATORS

Four different technologies for medical accelerators are compared in [2]. Although the comparisons are based on parameters beyond the optical design it is worth mentioning some of the results. First, it is concluded that the RCMS can achieve faster changes in the beam energy than any other accelerator, with a much smaller emittance, a moderate size, and adequate intensity for proton therapy.

Second, the energy of the beam can be easily varied in the required range of energies by just firing the extraction kicker at different times. In the cyclotron, the beam is extracted at maximum energy and reduced to the desired energy by a variable thickness energy degrader. In this sense, the delivered energy selection is variable, at the expense of a higher emittance from multiple scattering, larger gantries, and a higher radio-activation, which leads to higher shielding requirement.

TABLE IV
MAGNET POLE GAP AND MAGNET POLE TIP WIDTH FOR THREE
DIFFERENT FACILITIES

| Facility | Magnet Pole Gap [mm] | Magnet pole tip width [mm] |
|---|---|---|
| Loma Linda [8] | 50 | 200 |
| KUMPF [9] | 67 | 340 |
| RCMS [10] | 30 | 60 |

From the optical point of view, the RCMS has a very small dipole gap when compared to typical medical facilities already in operation like Loma Linda or facilities in design stage like Kyoto University Medical Proton Facility (KUMPF) (see Table IV). Since the dipole gap is directly correlated with the magnet size, the RCMS has smaller magnets when compared with the other facilities. For example, the transverse section of the dipole magnets that will be used in the KUMPF is $97 \times 80$ cm [9] while the transverse section of the dipole magnets in the RCMS is only $48 \times 24$ cm. Since the RCMS magnets are smaller than the ones employed in other facilities, the overall cost of the RCMS optical lattice is also expected to be lower.

## VIII. CONCLUSION

The overall optical lattice of the RCMS has been presented making special emphasis in the IBEFUMFO algorithm employed to facilitate the design. In particular, IBEFUMFO has made it possible to increase the spacing of the quadrupoles in the straight sections without increasing the beam size in the arcs and with a modest increase of the beam size in the straight sections. The increase of the spacing of the quadrupoles in the straight sections allows the RCMS to use small angles of extraction and also allow more space for the RF cavity, which reduces the cost of both the extraction system and the RF cavity.

IBEFUMFO has also been used in the design of the delivery optics, in particular in the extraction line where the algorithm has been used to match the incoming beta function of the ring to the beta functions of switch-yard, a beam line made out of FODO cells and which feeds all the beam lines going to the different rooms in the facility.

Particle trajectory studies inside the different magnet styles indicate that it is possible to use the 22.5° Chevron magnet as a C-type magnet and as a 6.5° magnet. This reduces the number of magnets styles from six styles to five styles.

Comparisons of this design with previous designs show significant savings in the injection and extraction systems, the power supplies and specially the RF system and the number of magnets to be employed in the facility.

Comparisons with other medical facilities show the technological advantages of the RCMS and lower costs, thank to the smaller magnets of the RCMS.

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